Proofs of Security Protocols
Symbolic Methods and Powerful Attackers

Charlie Jacomme,
supervised by Hubert Comon and Steve Kremer
June 30th, 2021
Introduction
Do I really need to introduce security and privacy?
One of the biggest lesson of my thesis
One of the biggest lesson of my thesis.

Privacy matters
Many people NEED privacy to live:

- Homosexuality is a crime in 69 countries.
- Citizens in authoritarian countries (journalist, political opponents).
- Discrimination (origins, health, religion,...) for loans, health insurances, employment...
- Uighurs currently tracked in China.
Many people NEED privacy to live:

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But what if I have nothing to hide?
1. If we don’t have privacy, people that need it can’t have it.
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2. Most people actually have something to hide.
   • Should my boss know what I do on my free time? Or what is the global income of my household?
   • Should my government know my political opinions?
   • Should my mail provider know my illness?
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   - Should my mail provider know my illness?

3. The simple fact of being watched changes unconsciously our behaviour.
   - Philosophical and sociological theories (Foucault, Deleuze, Guattari,…), and fictional examples (Orwell, Damosio,…).
Security and Privacy Matter!
Guarantees

Security and Privacy Matter!

For each possible use case, we should know exactly who can access what, whether it is a stranger, a government or a corporation.
Guarantees

Security and Privacy Matter!

Which guarantees, for which attacker?
Security and Privacy Matter!

Which *formal guarantees*, for which attacker?
The difficulty

Protocols

If any link of the chain is broken, everything is.
The difficulty

Primitives \quad Protocols

\[ x^2 \quad \iff \]

Hardware / mobile phone / laptop

OS

\ / windows / linux

Implementation / file_text_alt

Users / group

If any link of the chain is broken, everything is.
The difficulty

Implementation  Primitives  Protocols

\[ x^2 \]
The difficulty

<table>
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If any link of the chain is broken, everything is.
Second difficulty - How to model the attacker?

Symbolic Model  VS  Computational Model
## Second difficulty - How to model the attacker?

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Symbolic Model VS Computational Model

symbolic = fixed set of possible actions

symbolic = any program
Second difficulty - How to model the attacker?

Symbolic Model  VS  Computational Model

adversary = fixed set of possible actions  
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Automated proofs - strong assumptions
Second difficulty - How to model the attacker?

Symbolic Model VS Computational Model

*adversary = fixed set of possible actions*

*adversary = any program*

Automated proofs - strong assumptions

Hard automation - strong guarantees
My lines of work

The core of my PhD
Make it easier to prove protocols against attackers as powerful as possible.

• Make the symbolic model more precise (detailed threat models);
• enable proofs of compound protocols in the computational model:
  • compositional proofs,
  • mechanization,
  • proof automation.
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Summary of contributions - outline

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4. symbolic methods for deciding basic proof steps in computational proofs, formulated as problems on probabilistic programs.
   Not presented. (decidability of universal equivalence between programs over finite fields; library integrated into EasyCrypt and MaskVerif)
Make the symbolic model more precise
Second factor authentication
How to improve passwords (which are weak)
Use a second factor to confirm login, either a smartphone or a dedicated token.
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Use a second factor to confirm login, either a smartphone or a dedicated token.

Considered protocols:

- Google 2 Step (Verification code, Single Tap, Double Tap)
- FIDO’s U2F (Google, Facebook, Github, Dropbox,...)
Main ideas

A case study\(^1\) of Google 2 Step and FIDO’s U2F.

- Many different detailed threat models;
  - malware on the phone,
  - keylogger on the computer,
  - weak SMS channel,
  - ...
- model the full authentication system;
- completely automated analysis of all scenarios;
- simple, small modifications (adding info to display) that enhance security.

\(^1\)C. Jacomme and S. Kremer, CSF’18 & ACM TOPS
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\[\rightarrow\] 6 172 (non-redundant) scenarios analysed by PROVERIF

\(\text{\footnotesize in 8 minutes}\)

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Results

Pros of U2F

- A possibility of privacy.
- Strong protection against phishing.

Cons of U2F

- No feedback to the user, cannot verify what is validated.
- Not independent from the computer, risk of malwares.
An introduction to the BC logic
A protocol

\[ A \xrightarrow{\langle r, \text{sign}(r, key) \rangle} B \]

checks the signature \( <"ok", r> \)

Security properties:

• Authentication - whenever B accepts, the message that B received was sent by A.
A protocol

\[ A \xrightarrow{\langle r, \text{sign}(r, key) \rangle} B \]

| Checks the signature

Security property

- Authentication: whenever B accepts, the message that B received was sent by A.
Protocol and properties

A protocol

\[ A \quad \langle r, \text{sign}(r, \text{key}) \rangle \quad \overset{\text{checks the signature}}{\longrightarrow} \quad B \]

\[ \left< \text{"ok"}, r \right> \]
A protocol

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\leftarrow \langle \text{"ok"}, r \rangle
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A quick introduction to the BC logic

A protocol

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\begin{align*}
A \quad & \langle r, \text{sign}(r, \text{key}) \rangle \quad \rightarrow \quad B \\
& \quad \text{| Checks the signature} \\
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In BC

Protocols are modelled with sequences of terms:

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\phi_0 := \langle r, \text{sign}(r, key) \rangle
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\phi_1 := \phi_0, \quad \text{if} \left( \text{checks}(\text{snd}(g_0(\phi_0)), pk(key))) = \text{fst}(g_0(\phi_0)) \right) \text{then}
\]

\[
\langle \text{"ok"}, \text{fst}(g_0(\phi_0)) \rangle
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A quick introduction to the BC logic

A protocol

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\end{align*}
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How to reason on terms?
A first order logic built over a predicate that captures indistinguishability:

\[ t_1 \sim t_2 \]
A quick introduction to the BC logic

How to reason on terms?
A first order logic built over a predicate that captures indistinguishability:

\[ t_1 \sim t_2 \]

Indistinguishability

- A generic way to express all security properties.
- Any attacker can only distinguish between \( t_1 \) and \( t_2 \) with negligible probability.
A quick introduction to the BC logic

Axioms to model the assumptions about the cryptographic library.
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Axioms to model the assumptions about the cryptographic library.

**EUF-CMA**

\[
\text{checksign}(t, pk(key))) = m \Rightarrow \\
\bigvee_{\text{sign}(x, key) \in \text{St}(t)} (t = \text{sign}(x, key) \land x = m)
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A mechanized prover for the BC logic
Issues of the BC logic

Downsides of the BC logic

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- To perform proofs, we have to look at all executions of the protocol, i.e., all possible sequences of terms.
- Proofs are tedious to perform by hand.
- Proofs only for a bounded number of sessions.
Our contributions\(^2\)

- A meta-logic over BC, that allows to talk abstractly about executions of the protocol.

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---

signature sign, checksign, pk

abstract ok : message
abstract error : message

name key : message
name r : index -> message

channel c

process A(i:index) =
    out(c, <r(i), sign(r(i), key)>)

process B =
in(c,x);
if checksign(snd(x), pk(key)) = fst(x) then
    out(c,<fst(x), ok>)
else out(c, error)

system (! i A(i) | ! i B).
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goal auth :
  forall (i:index),
    (cond @B(i) =>
      exists (j:index),
      A(j) < B(i) && fst(input@B(i)) = fst(output@A(j)))

Proof.
signature sign, checksign, pk

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system (!_ A(i) | !_ B).

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Proof.
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Proof.
  simpl.
  expand cond@B(i).
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Proof.
  simpl.
  expand cond@B(i).
  euf M0.
  exists i1.
  Qed.
https://squirrel-prover.github.io/
A compositional framework inside the computational model
The goal
To be able to make the proof of a composed protocol as a composition of proofs:

• smaller,
• reusable,
• and modular proofs.
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Top-down vs bottom-up

- Prove components universally secure (UC), and combine them together.
- Split a protocol into multiple components, and prove them secure in the context.
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Composition?

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Limitations of the state of the art
Shared secrets and state passing and usability.
Our contributions

The composition framework\(^3\)

- Handles parallel and sequential composition;
  unlike Blanchet, CSF'18, or Brzuska et al., CCS'11

\(^3\)H. Comon, C. Jacomme and G. Scerri. CCS'20
Our contributions

The composition framework\textsuperscript{3}

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- allows to consider protocols with state passing and long term shared secrets;
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- allows to consider protocols with state passing and long term shared secrets;
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- allows to reduce the security of multiple sessions to the security of a single one;

- naturally translates to the BC logic, and allows for the first time to perform proofs for an unbounded number of sessions with this logic.

---

3H. Comon, C. Jacomme and G. Scerri. CCS’20
\( A \) is trying to break protocol \( P \), while also having access to \( Q \).
A classical proof technique

$A$ is trying to break protocol $P$, while also simulating $Q$. 
A is trying to break protocol $\mathcal{P}$, while also simulating $\mathcal{Q}$. 
A classical proof technique

\( A \) is trying to break protocol \( \mathcal{P} \), while also simulating \( \mathcal{Q} \).
A is trying to break protocol $\mathcal{P}$, while also simulating $\mathcal{Q}$. 
**The main idea**

If $A$ can simulate it, i.e., produce exactly all the same messages:

we remove $Q$ from the picture!
**The main idea**
If $\mathcal{A}$ can simulate it, i.e., produce exactly all the same messages:

we remove $Q$ from the picture!

**The difficulty**
If $P$ and $Q$ share some secret key, $\mathcal{A}$ cannot simulate messages which require key.
The main idea - example on downgrade attacks

\[ P = \text{version 1 of the previous protocol} \]

\[ A \xrightarrow{\text{sign}(\langle r, "v_1" \rangle, \text{key})} B \]

| Checks the signature | \[ <r,"ok"> \]

\[ Q = \text{version 2 of the previous protocol} \]

\[ A \xrightarrow{\text{sign}(\langle r, "v_2" \rangle, \text{key})} B \]

| Checks the signature | \[ <r,"ok"> \]
The main idea

Example for signatures

- $Q_{\text{key}}$ may produce $\text{sign}(< m, "v_1" >, key)$
- $P_{\text{key}}$ may produce $\text{sign}(< m', "v_2" >, key)$
The main idea

Example for signatures

- $Q_{\text{key}}$ may produce $\text{sign}(< m, "v_1" >, \text{key})$
- $P_{\text{key}}$ may produce $\text{sign}(< m', "v_2" >, \text{key})$

To prove $P$ while abstracting $Q$, the attacker must be able to produce $\text{sign}(< m', "v_1" >, \text{key})$. 
The main idea

Example for signatures

- $Q_{\text{key}}$ may produce $\text{sign}(<m, "v_1">, \text{key})$
- $P_{\text{key}}$ may produce $\text{sign}(<m', "v_2">, \text{key})$

To prove $P$ while abstracting $Q$, the attacker must be able to produce $\text{sign}(<m', "v_1">, \text{key})$.

$\leftarrow$ We may give an oracle to the attacker, allowing to obtain $\text{sign}(<m', "v_1">, \text{key})$ but not $\text{sign}(<m, "v_2">, \text{key})$. 
$A$ is trying to break protocol $\mathcal{P}$, while simulating $Q$ thanks to oracle $\mathcal{O}$. 

Diagram:

- $\mathcal{P}_{\text{key}}$
- $B^\mathcal{O}$
- $A$
$A$ is trying to break protocol $\mathcal{P}$, while simulating $Q$ thanks to oracle $O$. 

$\mathcal{P}_{\text{key}}$
The main idea

$A$ is trying to break protocol $\mathcal{P}$, while simulating $Q$ thanks to oracle $\mathcal{O}$. 

\[ P_{\text{key}} \]

\[ A'\mathcal{O} \]
The main idea

$A$ is trying to break protocol $\mathcal{P}$, while simulating $Q$ thanks to oracle $\mathcal{O}$. 

$\mathcal{P}_{\text{key}}$ 

$A^\mathcal{O}$
Simulatability

\[ \nu_{\text{key}}. Q_{\text{key}} \text{ is } \mathcal{O}\text{-simulatable} \iff \]
there exists a PPT \( \mathcal{A}^\mathcal{O} \) which, for any fixed value of \( \text{key} \),
produces exactly the same distribution as \( Q_{\text{key}} \).
Simulatability

\[ \nu \text{key} \cdot Q_{\text{key}} \text{ is } O\text{-simulatable} \]

iff

there exists a PPT \( A^{O} \) which, for any fixed value of \( \text{key} \),
produces exactly the same distribution as \( Q_{\text{key}} \)

A protocol

\[ Q := \ldots \]

\[ \text{out}(\text{sign}(\langle \text{mess}, \text{“v1”} \rangle, \text{key}))) \]
Simulatability

\[ \nu \text{key. } Q_{\text{key}} \text{ is } O\text{-simulatable} \]

iff

there exists a PPT \( A^O \) which, for any fixed value of \( \text{key} \),
produces exactly the same distribution as \( Q_{\text{key}} \)

A protocol

\[ Q := \ldots \]

\[ \text{out}(\text{sign}(\langle \text{mess, "v"}_1 \rangle, \text{key}))) \]

Signing oracle

\[ O^\text{sign}_{\text{key}} : \text{input}(m) \]

\[ \text{output}(\text{sign}(\langle m, "v"_1 \rangle, \text{key}))) \]
Generic signing oracles

$T$ signing oracle

$O_{T,sk}^{\text{sign}} : \text{input}(m)$

if $T(m)$ then

output($\text{sign}(m, sk)$)
Generic signing oracles

**T signing oracle**

\[ \mathcal{O}_{T,sk}^{\text{sign}} : \text{input}(m) \]

**if** \( T(m) \) **then**

**output**(\( \text{sign}(m, sk) \))

**T-EUF-CMA**

\[ \text{checksign}(t, pk(sk)) \models m \Rightarrow \]

\[ T(m) \]

\[ \forall_{\text{sign}(x, sk) \in \text{St}(t)} (t \models \text{sign}(x, sk) \land x \models m) \]
Conclusions about compositions

- Used as a proof of concept on SSH;
- proofs close to the classical ones;
- mechanizable.

→ It was easy to extend Squirrel to support the generic axioms!
Conclusion
Summary of contributions

1. a methodology to analyze protocols in the symbolic model, but making the attacker as strong as possible, with a case study on multi-factor authentication;
   - C. Jacomme and S. Kremer. CSF’18 & ACM TOPS
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What’s next

Modularity
Apply/extend the composition framework to more complex protocols and properties.
(e-voting protocols, forward secrecy for key-exchanges)
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**Automation**
Improve the automation,

- at the high-level, in the Squirrel Prover; (e.g. with SMT solvers)
- at the low-level, through SolvEq.

Collaboration
There will not be one tool to rule them all. Use each for what it does best and combine formally the guarantees.
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There will not be one tool to rule them all. Use each for what it does best and combine formally the guarantees.
Privacy
Privacy

• It matters.
Privacy

- It matters.
- Most people, corporation and states don’t care about it.
Privacy

• It matters.
• Most people, corporation and states don’t care about it.

I would like to try to do something about that…
Some appendixes
Composition on an example
Main intuition

Basic Theorem Example - Parallel Composition
Given two protocols $P$ and $Q$, with $\bar{n} = N(P) \cap N(Q)$, if:

- $\nu \bar{n}.Q$ is $O$-simulatable;
- $A \models P \sim P'$;
- the axioms $A$ are sound for machines with access to $O$.

Then $A \models P \parallel Q \sim P' \parallel Q$. 
A small DDH example

Signed DDH

\[ A(a, skA) \]  \quad \leftarrow \quad \text{sign}(g^a, skA) \quad \rightarrow \quad \text{sign}(\langle g^a, g^b \rangle, skB) \quad \leftarrow \quad x_B = g^a \]

\[ x_A = g^b \]

\[ k_A = x_A^a \]

\[ B(b, skB) \]  \quad \rightarrow \quad \text{sign}(\langle g^a, g^b \rangle, skA) \quad \leftarrow \quad k_B = x_B^b \]
A small DDH example

The security property:

\[ \| i \leq N (A(a_i, skA); \text{out}(k_A)) \| B(b_i, skB); \text{out}(k_B)) \sim \]

\[ \| i \leq N - 1 (A(a_i, skA); \text{out}(k_A)) \| B(b_i, skB); \text{out}(k_B)) \]

\[ \| A(a_N, skA); \text{if } x_A = g^{b_N} \text{ then } \text{out}(k_{N,N}) \]

\[ \text{else if } x_A \notin \{g^{b_i}\}_{1 \leq i \leq N} \text{ then } \perp \]

\[ \| B(b_N, skB); \text{if } x_B = g^{a_N} \text{ then } \text{out}(k_{N,N}) \]

\[ \text{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \text{ then } \perp \]
A small DDH example

The final security property:
Let’s assume the attacker can simulate

\[ \|_{i \leq N-1} (A(a_i, skA); \text{out}(k_A)) \| B(b_i, skB); \text{out}(k_B)) \]

...
A small DDH example

The final security property:
Let’s assume the attacker can simulate

$$\|_{i \leq N-1} (A(a_i, skA); \text{out}(k_A)\| B(b_i, skB); \text{out}(k_B))$$

We can simply prove:

$$A(a_N, skA); \text{out}(k_A)\| B(b_N, skB); \text{out}(k_B)$$

$$\sim$$

$$A(a_N, skA); \text{if } x_A = g^{bN} \text{ then out}(k_{N,N})$$

else if $$x_A \notin \{g^{b_i}\}_{1 \leq i \leq N}$$ then ⊥

$$\| B(b_N, skB); \text{if } x_B = g^{aN} \text{ then out}(k_{N,N})$$

else if $$x_B \notin \{g^{a_i}\}_{1 \leq i \leq N}$$ then ⊥
A small DDH example

The final security property:
Let’s assume the attacker can simulate

\[ \| \overset{i \leq N-1}{(A(a_i, skA); \text{out}(k_A)) \| B(b_i, skB); \text{out}(k_B))} \]

We can simply prove:

\[ A(a_N, skA); \text{out}(k_A) \| B(b_N, skB); \text{out}(k_B) \sim \]

\[ A(a_N, skA); \text{if } x_A = g^{b_N} \text{ then out}(k_{N,N}) \]
\[ \text{else if } x_A \notin \{g^{b_i}\}_{1 \leq i \leq N} \text{ then } \perp \]
\[ \| B(b_N, skB); \text{if } x_B = g^{a_N} \text{ then out}(k_{N,N}) \]
\[ \text{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \text{ then } \perp \]

\[ \leftrightarrow \text{How to simulate the } N - 1 \text{ sessions?} \]
Simulating the sessions

What must the attacker be able to produce?
He must be able to start some $A$:

\[
\forall 1 \leq i \leq N - 1. \ sign(g^a_i, skA)
\]
What must the attacker be able to produce?

He must be able to start some $A$:

$$\forall 1 \leq i \leq N - 1. \text{sign}(g^{a_i}, skA)$$

And for any DDH share $r$ he receives, he should be able to produce:

- $$\forall 1 \leq i \leq N - 1. \text{sign}(<g^{a_i}, r>, skA)$$
Simulating the sessions

What must the attacker be able to produce?
He must be able to start some $A$:

$$\forall 1 \leq i \leq N - 1. \; \text{sign}(g^{a_i}, skA)$$

And for any DDH share $r$ he receives, he should be able to produce:

- $\forall 1 \leq i \leq N - 1. \; \text{sign}(< g^{a_i}, r >, skA)$
- $\forall 1 \leq i \leq N - 1. \; \text{sign}(< r, g^{b_i} >, skB)$
Generic signing oracles

**T signing oracle**

\[ O_{T,sk}^{\text{sign}} : \text{input}(m) \]

\[ \text{if } T(m) \text{ then} \]

\[ \text{output}(\text{sign}(m, sk)) \]

Give the attacher access to \( O_{T,sk}^{\text{sign}}, sk_A \) and \( O_{T,sk}^{\text{sign}}, sk_B \) with:

\[ T(m) = \text{true} \Leftrightarrow \exists 1 \leq i \leq N - 1, r. \]

\[ m = ga_i \]

\[ m = < ga_i, r > \]

\[ m = < r, gb_i > \]
Generic signing oracles

**T signing oracle**

\[ \mathcal{O}_{T,sk}^{\text{sign}} : \text{input}(m) \]

**if** \( T(m) \) **then**

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\[
T(m) = \text{true} \iff \exists 1 \leq i \leq N - 1, r. \begin{cases} 
m = g^{a_i} \\
m = \langle g^{a_i}, r \rangle \\
m = \langle r, g^{b_i} \rangle
\end{cases}
\]
Generic signing oracles

T signing oracle

\( O_{T,sk}^{\text{sign}} \) : input \((m)\)

if \( T(m) \) then

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\[
T(m) = \text{true} \iff \exists 1 \leq i \leq N - 1, r. \left\{ \begin{array}{l}
    m = g^{a_i} \\
    m = \langle g^{a_i}, r \rangle \\
    m = \langle r, g^{b_i} \rangle 
\end{array} \right.
\]

\( \leftrightarrow \) How to make the proof for such attackers?
**T-EUF-CMA**

For any computable function $T$, for all terms $t$ such that $sk$ only appears in key position:

\[
\text{checksing}(t, pk(sk)) \Rightarrow \\
T(\text{getmess}(t)) \\
\bigvee_{\text{sign}(x,sk) \in \text{st}(t)}(t \doteq \text{sign}(x, sk)) \\
\sim \text{true}
\]
The final proof

Assumption

\[ \text{checks}(t, pk(sk)) \Rightarrow \exists 1 \leq i \leq N - 1, r. \ \text{getmess}(t) \in \{ g^{a_i}, < g^{a_i}, r >, < r, g^{b_i} > \} \]

\[ \bigvee_{\text{sign}(x, sk) \in \text{St}(t)} (t \doteq \text{sign}(x, sk)) \]

\[ \sim \text{true} \]
The final proof

Assumption

\[
\text{checksing}(t, pk(sk)) \Rightarrow \\
\exists 1 \leq i \leq N - 1, r. \ \text{getmess}(t) \in \{g^{ai}, < g^{ai}, r >, < r, g^{bi} >\} \\
\bigvee_{\text{sign}(x, sk) \in \text{St}(t)} (t = \text{sign}(x, sk)) \\
\sim true \\
\wedge \ \text{DDH}: g^{aN}, g^{bN}, g^{aNbN} \sim g^{aN}, g^{bN}, k_{N,N}
\]
Goal

\[
A(a_N, sk_A); \text{out}(k_A) \parallel B(b_N, sk_B); \text{out}(k_B)
\]

\[
\sim
\]

\[
A(a_N, sk_A); \text{if } x_A = g^{b_N} \text{ then out}(k_{N,N})
\]

\[
\text{else if } x_A \notin \{g^{b_i}\}_{1 \leq i \leq N} \text{ then } \bot
\]

\[
\parallel B(b_N, sk_B); \text{if } x_B = g^{a_N} \text{ then out}(k_{N,N})
\]

\[
\text{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \text{ then } \bot
\]
Synchronization

\[ A(a_N, skA); \text{ if } x_A = g^{b_N} \text{ then out}(g^{a_N b_N}) \]
\[ \text{ else if } x_A \not\in \{g^{b_i}\}_{1 \leq i \leq N} \text{ then out}(x_A^{a_N}) \]
\[ \| B(b_N, skB); \text{ if } x_B = g^{a_N} \text{ then out}(g^{a_N b_N}) \]
\[ \text{ else if } x_B \not\in \{g^{a_i}\}_{1 \leq i \leq N} \text{ then out}(x_B^{b_N}) \]
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\[ A(a_N, skA); \text{ if } x_A = g^{b_N} \text{ then out}(k_{N,N}) \]
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The final proof

**Synchronization**

\[ A(a_N, \text{sk}A); \textbf{if } x_A = g^{b_N} \textbf{ then out}(g^{a_N b_N}) \]

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\[ \text{∥ } B(b_N, \text{sk}B); \textbf{if } x_B = g^{a_N} \textbf{ then out}(g^{a_N b_N}) \]

\[ \text{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \textbf{ then out}(x_B^{b_N}) \]

\[ \sim \]

\[ A(a_N, \text{sk}A); \textbf{if } x_A = g^{b_N} \textbf{ then out}(k_{N,N}) \]

\[ \text{else if } x_A \notin \{g^{b_i}\}_{1 \leq i \leq N} \textbf{ then ⊥} \]

\[ \text{∥ } B(b_N, \text{sk}B); \textbf{if } x_B = g^{a_N} \textbf{ then out}(k_{N,N}) \]

\[ \text{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \textbf{ then ⊥} \]
The final proof

Synchronization

\[
A(a_N, skA); \text{ if } x_A = g^{b_N} \text{ then out}(g^{a_N b_N}) \\
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\text{ else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \text{ then out}(x_B^{b_N}) \\
\sim \\
A(a_N, skA); \text{ if } x_A = g^{b_N} \text{ then out}(k_{N,N}) \\
\text{ else if } x_A \notin \{g^{b_i}\}_{1 \leq i \leq N} \text{ then } \bot \\
\parallel B(b_N, skB); \text{ if } x_B = g^{a_N} \text{ then out}(k_{N,N}) \\
\text{ else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \text{ then } \bot
\]
The final proof

**Synchronization**
Proof steps Split the conditionals into four cases and,
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1. use DDH to show indistinguishability,
**Synchronization**

Proof steps Split the conditionals into four cases and,

1. use DDH to show indistinguishability,

2. use T-EUF-CMA, to show that $x_A \notin \{g^{b_i}\}_{1 \leq i \leq N}$ is never true (e.g, \bot unreachable),
The final proof

**Synchronization**
Proof steps Split the conditionals into four cases and,

1. use DDH to show indistinguishability,
2. use T-EUF-CMA, to show that $x_A \not\in \{g^{bi}\}_{1 \leq i \leq N}$ is never true (e.g, ⊥ unreachable),
3. similar to (2);
4. similar to (2);
Formal composition theorems
A core theorem

Composition without replication
Let $C[_1, \ldots, _n]$ be a context such that the variable $k_i$ is bound in each hole $-_i$ and $P_1(x), \ldots, P_n(x)$ be parametrized protocols, such that all channels are disjoint. Given an oracle $O$, with $\overline{n} \supset \mathcal{N}(C) \cap \mathcal{N}(P_1, \ldots, P_n)$, if, with $k_1', \ldots, k_n'$ fresh names,

1. $C[\text{out}(1, k_1), \ldots, \text{out}(n, k_n)] \approx_O C[\text{out}(1, k_1'), \ldots, \text{out}(n, k_n')]$

2. $\nu \overline{n}.\text{in}(x).P_1(x) \parallel \ldots \parallel \text{in}(x).P_n(x)$ is $O$-simulatable

Then $C[P_1(k_1), \ldots, P_n(k_n)] \approx_O C[P_1(k_1'), \ldots, P_n(k_n')]$
Unbounded parallel Composition

Let $O_r$ be an oracle and $Ax$ a set of axioms both parametrized by a sequence of names $\bar{s}$. Let $\bar{p}$ be a sequence of shared secrets, $P(x), R(x, y, z)$ and $Q(x, y)$ be parametrized protocols. If we have, for a sequence of names $\text{lsid}$ and any integers $n$, if with $\bar{s} = \text{lsid}_1, \ldots, \text{lsid}_n$ $n$ copies of $\text{lsid}$:

1. $\forall 1 \leq i \leq n, \nu \bar{p}. t_{R(\bar{p}, \text{lsid}_i, s)}$ is $O_r$ simulatable.
2. $Ax$ is $O_r$ sound.
3. $Ax \models t_{P(\bar{p})} \sim t_{Q(\bar{p}, s)}$

Then, for any integer $n$:

$$P(\bar{p}) \parallel !_n R(\bar{p}, \text{lsid}, \bar{s}) \cong Q(\bar{p}, \bar{s}) \parallel !_n R(\bar{p}, \text{lsid}, \bar{s})$$
A core theorem

Unbounded parallel Composition
Let $O_r$ be an oracle and $Ax$ a set of axioms both parametrized by a sequence of names $\bar{s}$. Let $\bar{p}$ be a sequence of shared secrets, $P(\bar{x}, \bar{y})$ and $Q(\bar{x}, \bar{y}, \bar{z})$ be parametrized protocols. If we have, for sequences of names $\bar{lsid}_p, \bar{lsid}_q$ and any integers $n$, if with $\bar{s} = \bar{lsid}_p,1, \ldots, \bar{lsid}_p,n, \ldots, \bar{lsid}_q,n$ sequences of copies of $\bar{lsid}_p, \bar{lsid}_q$

1. $\forall 1 \leq i \leq n, \nu \bar{p}.P(\bar{p}, \bar{lsid}_p,i) \Rightarrow O_r$ simulatable.

2. $\forall 1 \leq i \leq n, \nu \bar{p}.Q(\bar{p}, \bar{lsid}_q,i, \bar{s}) \Rightarrow O_r$ simulatable.

3. $Ax$ is $O_r$ sound.

4. $Ax \models P(\bar{p}, \bar{lsid}_p) \sim Q(\bar{p}, \bar{lsid}_q, \bar{s})$

Then, for any integers $n$:

$$!_n P(\bar{p}, \bar{lsid}_p) \equiv O !_n Q(\bar{p}, \bar{s}, \bar{lsid}_q)$$