Proofs of Security Protocols

Symbolic Methods and Powerful Attackers

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supervised by Hubert Comon and Steve Kremer

June 30th, 2021

Introduction

Introduction?

Do I really need to introduce security and privacy?

Introduction

One of the biggest lesson of my thesis

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Privacy matters

Privacy

Many people NEED privacy to live:

- Homosexuality is a crime in 69 countries.
- Citizens in authoritarian countries (journalist, political opponents).
- Discrimination (origins, health, religion,. . .) for loans, health insurances, employment. . .
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But what if I have nothing to hide?

Nothing to hide. . .

1. If we don't have **privacy**, people that need it can't have it.

Nothing to hide...

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2. Most people actually have something to hide.
   - Should my boss know what I do on my free time? Or what is the global income of my household?
   - Should my government know my political opinions?
   - Should my mail provider know my illness?

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   - Should my boss know what I do on my free time? Or what is the global income of my household?
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3. The simple fact of being watched changes unconsciously our behaviour.
   - Philosophical and sociological theories (Foucault, Deleuze, Guattari,. . .), and fictional examples (Orwell, Damosio,. . .)

Guarantees

Security and Privacy Matter !

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For each possible use case, we should know exactly who can access what, whether it is a stranger, a government or a corporation.

Guarantees

Security and Privacy Matter !

Which guarantees, for which attacker?

Security and Privacy Matter !

Which formal guarantees, for which attacker?

The difficulty

Protocols

Primitives Protocols

x^2 ⇄

The difficulty

Implementation | Primitives | Protocols

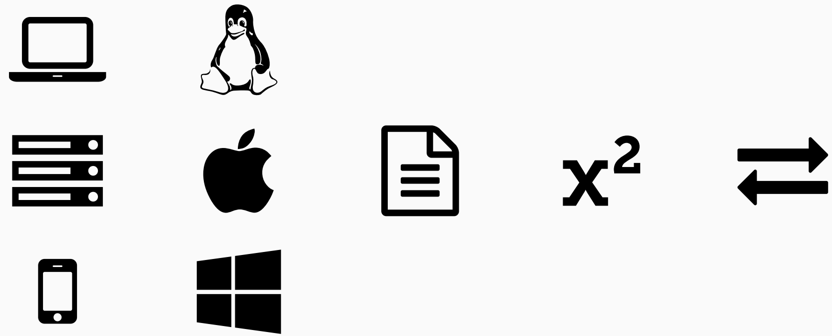
x^2

The difficulty

OS Implementation Primitives Protocols

The difficulty

Hardware OS Implementation Primitives Protocols



The difficulty

Hardware | OS | Implementation | Primitives | Protocols | Users

The difficulty

If any link of the chain is broken, everything is.

Hardware	OS	Implementation	Primitives	Protocols	Users

The difficulty

Hardware | OS | Implementation | Primitives | Protocols | Users

Second difficulty - How to model the attacker ?

Symbolic Model VS Computational Model

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Symbolic Model	Computational Model
adversary = fixed set of possible actions	*adversary = any program*

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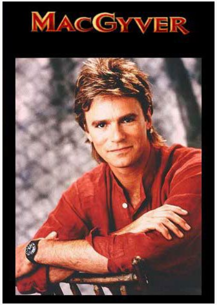

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Automated proofs - strong assumptions

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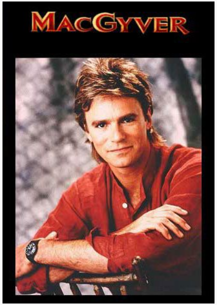

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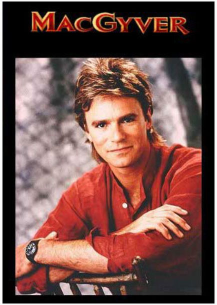

Hard automation - strong guarantees

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- Make the symbolic model more precise (detailed threat models);
- enable proofs of compound protocols in the computational model:
  - compositional proofs,
  - mechanization,
  - proof automation.

Summary of contributions - outline

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   Part III - presentation of the framework
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4. symbolic methods for deciding basic proof steps in computational proofs, formulated as problems on probabilistic programs.
   Not presented. (decidability of universal equivalence between programs over finite fields; library integrated into EasyCrypt and MaskVerif)

Make the symbolic model more precise

Second factor authentication

A case study : Second Factor authentication

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Use a second factor to confirm login, either a smartphone or a dedicated token.

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Considered protocols:

- Google 2 Step (Verification code, Single Tap, Double Tap)
- FIDO's U2F (Google, Facebook, Github, Dropbox,...)

Main ideas

A case study[1] of Google 2 Step and FIDO's U2F.

- Many different detailed threat models;
    - malware on the phone,
    - keylogger on the computer,
    - weak SMS channel,
    - ...
- model the full authentication system;
- completely automated analysis of all scenarios;
- simple, small modifications (adding info to display) that enhance security.

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→ 6 172 (non-redundant) scenarios analysed by PROVERIF in 8 minutes

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Threat Scenarios				g2V$^{\text{fpr}}$	g2V$^{\text{dis}}$	g2OT$^{\text{dis}}$	g2DT$^{\text{dis}}$
PH				✔	✔	✔	✔
PH	FS			✖	✗✓✓	✖	✗✓✓
PH	FS	$\mathcal{M}^{\textbf{t-tls}}_{\textbf{io}:\mathcal{RO}}$		✖	✖	✖	✗✓✗
PH	FS	$\mathcal{M}^{\textbf{t-usb}}_{\textbf{in}:\mathcal{RO}}$		✖	✖	✖	✗✓✓
PH	FS	$\mathcal{M}^{\textbf{t-dis}}_{\textbf{io}:\mathcal{RW}}$		✖	✗✓✓	✖	✖
		$\mathcal{M}^{\textbf{t-tls}}_{\textbf{io}:\mathcal{RO}}$		✔	✔	✓✓✓	✔
		$\mathcal{M}^{\textbf{t-usb}}_{\textbf{in}:\mathcal{RO}}$		✔	✔	✓✓✓	✔
		$\mathcal{M}^{\textbf{t-tls}}_{\textbf{io}:\mathcal{RW}}$		✗✓✗	✓✓✗	✓✓✗	✓✓✗
		$\mathcal{M}^{\textbf{t-usb}}_{\textbf{in}:\mathcal{RW}}$		✓✓✗	✓✓✗	✓✓✗	✓✓✗
		$\mathcal{M}^{\textbf{t-usb}}_{\textbf{in}:\mathcal{RW}}$	$\mathcal{M}^{\textbf{t-tls}}_{\textbf{io}:\mathcal{RO}}$	✓✓✗	✓✓✗	✓✓✗	✓✓✗
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Pros of U2F

- A possibility of privacy.
- Strong protection against phishing.

Cons of U2F

- No feedback to the user, cannot verify what is validated.
- Not independent from the computer, risk of malwares.

An introduction to the BC logic

Protocol and properties

A protocol

$$A \xrightarrow{\langle r,\, \mathtt{sign}(r, key)\rangle} B$$

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Security property

- Authentication - whenever B accepts, the message that B received was sent by A.

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In BC

Protocols are modelled with sequences of terms:

$$\phi_0 := \langle r,\ \mathtt{sign}(r, key)\rangle$$

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$$\phi_1 := \phi_0, \begin{array}{l} \textbf{if } (\ \mathtt{checksign}(\mathtt{snd}(g_0(\phi_0)), pk(key))) = \mathtt{fst}(g_0(\phi_0)) \textbf{ then} \\ \quad < \text{“}ok\text{”}, \mathtt{fst}(g_0(\phi_0)) > \end{array}$$

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Attacker inputs represented with non instantiated function symbol

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How to reason on terms?
A first order logic built over a predicate that captures indistinguishability:

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Indistinguishability

- A generic way to express all security properties.
- Any attacker can only distinguish between t_1 and t_2 with negligible probability.

A quick introduction to the BC logic

Axioms to model the assumptions about the cryptographic library.

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EUF-CMA

$$\begin{aligned} &\mathtt{checksign}(t, pk(key)) = m \Rightarrow \\ &\qquad \bigvee_{\mathsf{sign}(x, key) \in \mathtt{St}(t)} (t = \mathsf{sign}(x, key) \wedge x = m) \end{aligned}$$

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A mechanized prover for the BC logic

Downsides of the BC logic

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- Proofs only for a bounded number of sessions.

Our contributions[2]

- A meta-logic over BC, that allows to talk abstractly about executions of the protocol.

[2] D. Baelde, S. Delaune, C. Jacomme, A. Koutsos, S. Moreau. S&P'21

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The Squirrel Prover

[2] D. Baelde, S. Delaune, C. Jacomme, A. Koutsos, S. Moreau. S&P'21

```
signature sign,checksign,pk

abstract ok : message
abstract error : message

name key : message
name r : index -> message

channel c

process A(i:index) =
  out(c, <r(i),sign(r(i),key)>)

process B =
  in(c,x);
  if checksign(snd(x),pk(key)) = fst(x) then
    out(c,<fst(x),ok>)
  else out(c,error)

system (!_i A(i) | !_i B).
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-:**- auth.sp   Top (22,0)   (squirrel script +2 [1])

Applications Emplacements Emacs lun. 15:44

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goal auth :
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        && fst(input@B(i)) = fst(output@A(j)).
Proof.
```

-:**- auth.sp Top (33,0) (squirrel script +2 [1] Scripting)

```
[goal> Focused goal (1/1):
System: default/both
----------------------------------------
forall (i:index),
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U:%*- *goals* All (8,0) (squirrel goals +2 [2])

U:%%- *response* All (1,0) (squirrel response [3])

Applications Emplacements Emacs lun. 15:45 en 100%

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 simpl.
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System: default/both
Variables: i:index
H0: cond@B(i)
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[goal> Focused goal (1/1):
System: default/both
Variables: i,i1:index
M0: checksign(snd(input@B(i)),pk(key)) = fst(input@B(i))
M1: r(i1) = fst(input@B(i))
T0: A(i1) < B(i)
----------------------------------------
exists (j:index), (A(j) < B(i) && fst(input@B(i)) = fst(output@A(j)))
```

```
U:%*-  *goals*      All (1,0)      (squirrel goals +2 [2] )
```

```
U:%%-  *response*   All (1,0)      (squirrel response [3] )
```

Applications Emplacements Emacs lun. 15:45 9 en 100%

```
signature sign,checksign,pk

abstract ok : message
abstract error : message

name key : message
name r : index -> message

channel c

process A(i:index) =
  out(c, <r(i),sign(r(i),key)>)

process B =
  in(c,x);
  if checksign(snd(x),pk(key)) = fst(x) then
    out(c,<fst(x),ok>)
  else out(c,error)

system (!_i A(i) | !_i B).

goal auth :
  forall (i:index),
    cond@B(i) =>
      exists (j:index),
         A(j) < B(i)
         && fst(input@B(i)) = fst(output@A(j)).
Proof.
 simpl.
 expand cond@B(i).
 euf M0.
 exists i1.
Qed.
```

```
[goal> Goal auth is proved
```

U:%%- *goals* All (1,0) (squirrel goals +2 [2])

-:**- auth.sp All (33,0) (squirrel script +2 [1] Scripting)

U:%%- *response* All (1,0) (squirrel response [3])

More details?

https://squirrel-prover.github.io/

A compositional framework inside the computational model

Composition?

The goal
To be able to make the proof of a composed protocol as a composition of proofs:

- smaller,
- reusable,
- and modular proofs.

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- Split a protocol into multiple components, and prove them secure in the context.

Limitations of the state of the art
Shared secrets and state passing and usability.

Our contributions

The composition framework[3]

- Handles parallel and sequential composition;

  unlike Blanchet, CSF'18, or Brzuska et al., CCS'11

[3] H. Comon, C. Jacomme and G. Scerri. CCS'20

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- allows to consider protocols with state passing and long term shared secrets;

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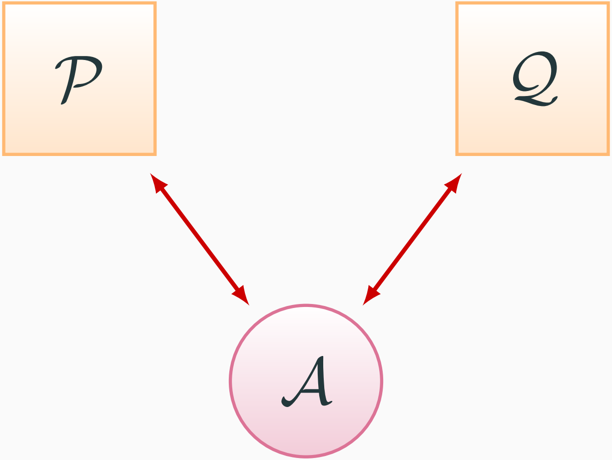
- allows to reduce the security of multiple sessions to the security of a single one;
- naturally translates to the BC logic, and allows for the first time to perform proofs for an unbounded number of sessions with this logic.

[3] H. Comon, C. Jacomme and G. Scerri. CCS'20

A classical proof technique

$\mathcal{A}$ is trying to break protocol $\mathcal{P}$, while also having access to $\mathcal{Q}$.



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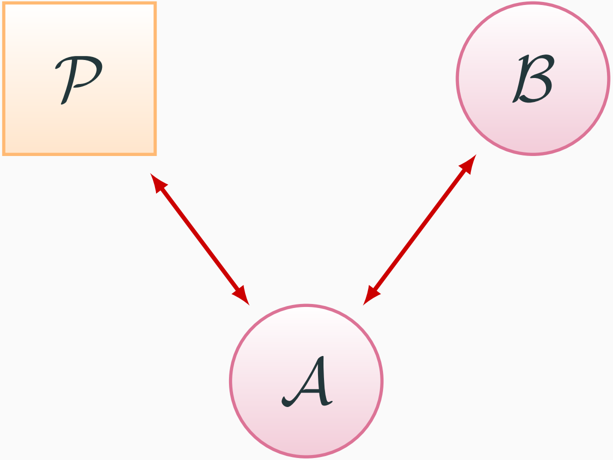

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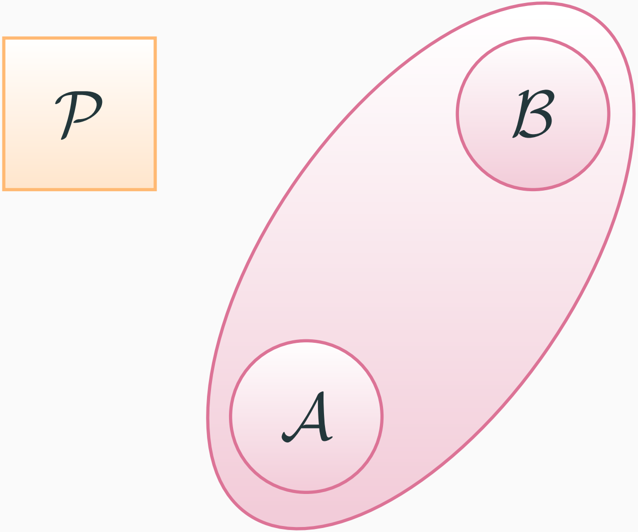

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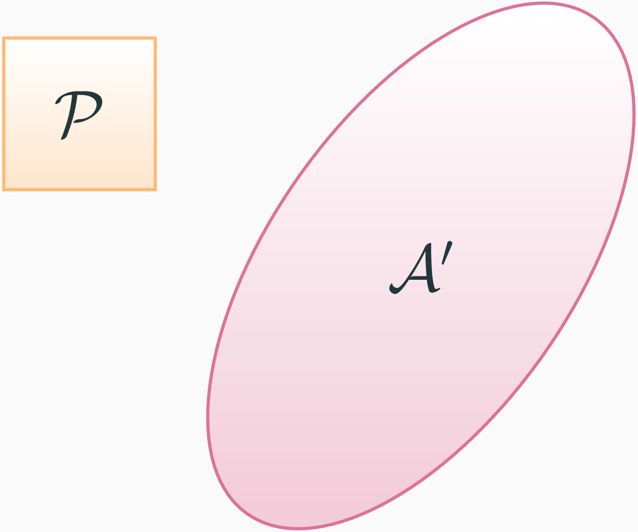

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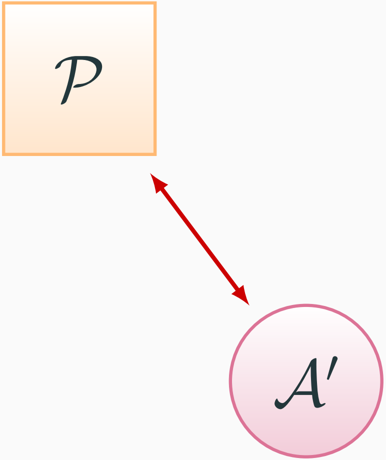

Simulation

The main idea

If $\mathcal{A}$ can simulate it, i.e., produce exactly all the same messages:

we remove Q from the picture!

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we remove Q from the picture!

The difficulty
If P and Q share some secret key, $\mathcal{A}$ cannot simulate messages which require key.

The main idea - example on downgrade attacks

P = version 1 of the previous protocol

$$A \xrightarrow{\texttt{sign}(\langle r, \text{``v}_1\text{''}\rangle, key)} B$$

| Checks the signature

$$\xleftarrow{<r, \text{``}ok\text{''}>}$$

Q = version 2 of the previous protocol

$$A \xrightarrow{\texttt{sign}(\langle r, \text{``v}_2\text{''}\rangle, key)} B$$

| Checks the signature

$$\xleftarrow{<r, \text{``}ok\text{''}>}$$

The main idea

Example for signatures

- $\mathcal{Q}_{key}$ may produce $\texttt{sign}(< m, \text{“v}_1\text{”} >, key)$
- $\mathcal{P}_{key}$ may produce $\texttt{sign}(< m', \text{“v}_2\text{”} >, key)$

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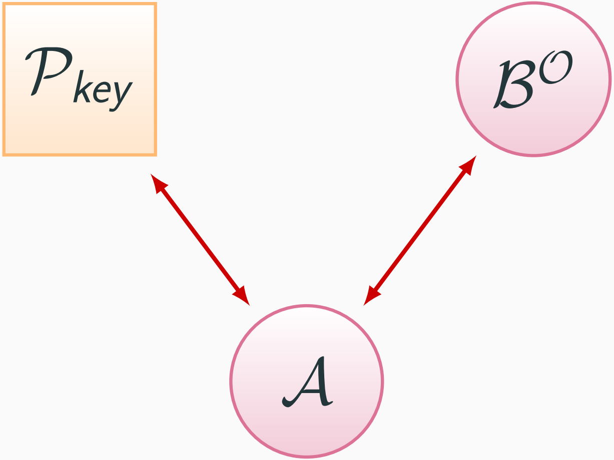
To prove $\mathcal{P}$ while abstracting $\mathcal{Q}$, the attacker must be able to produce $\texttt{sign}(< m', \text{“v}_1\text{”} >, key)$.

$\hookrightarrow$ We may give an oracle to the attacker, allowing to obtain $\texttt{sign}(< m', \text{“v}_1\text{”} >, key)$ but not $\texttt{sign}(< m, \text{“v}_2\text{”} >, key)$

The main idea

$\mathcal{A}$ is trying to break protocol $\mathcal{P}$, while simulating $\mathcal{Q}$ thanks to oracle $\mathcal{O}$.



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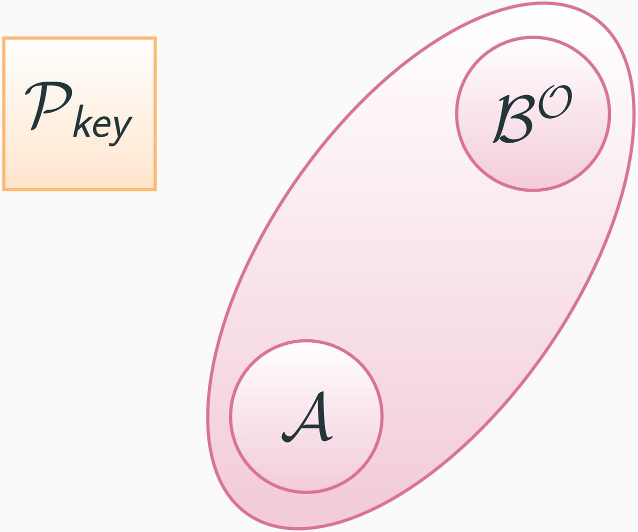

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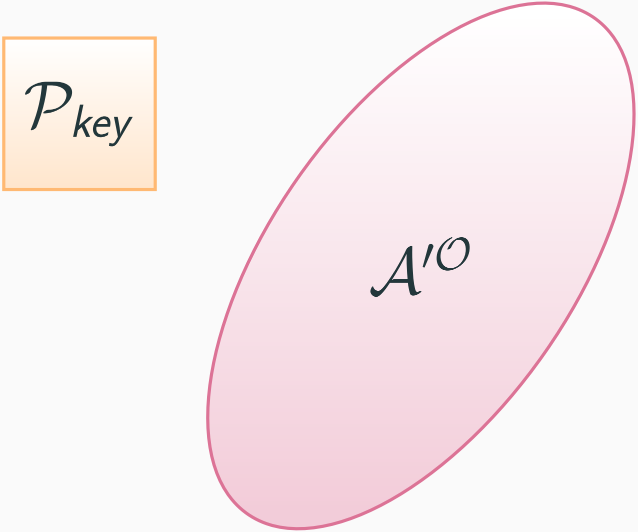

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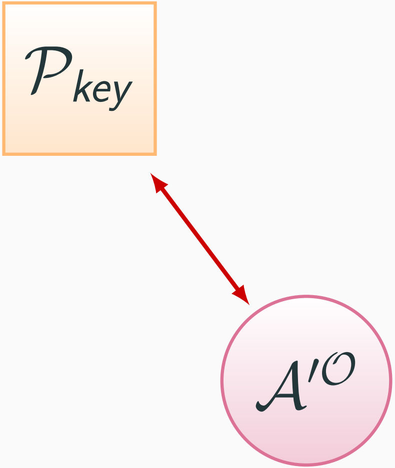

Simulatability

$\nu key.\mathcal{Q}_{key}$ is $\mathcal{O}$-simulatable

iff

there exists a PPT $\mathcal{A}^{\mathcal{O}}$ which, for any fixed value of *key*,

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$$\begin{aligned} Q := &\ \dots \\ &\ \textbf{out}(\text{sign}(\langle mess, \text{``v}_1\text{''}\rangle, key))) \end{aligned}$$

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Signing oracle

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T signing oracle

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$$\texttt{checksign}(t, pk(sk)) \doteq m \Rightarrow$$
$$T(m)$$
$$\bigvee_{\text{sign}(x,sk) \in \texttt{St}(t)} (t \doteq \text{sign}(x, sk) \wedge x \doteq m)$$

Conclusions about compositions

- Used as a proof of concept on SSH;
- proofs close to the classical ones;
- mechanizable.

$\hookrightarrow$ It was easy to extend Squirrel to support the generic axioms!

Conclusion

Summary of contributions

1. a methodology to analyze protocols in the symbolic model, but making the attacker as strong as possible, with a case study on multi-factor authentication;
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4. symbolic methods for deciding basic proof steps in computational proofs, formulated as problems on probabilistic programs;
    - G. Barthe, X. Fan, J. Gancher, B. Grégoire, C. Jacomme, and E. Shi. CCS'18
    - G. Barthe, B. Grégoire, C. Jacomme, S. Kremer, and P-Y. Strub. CSF'19
    - G. Barthe, C. Jacomme, and S. Kremer. LICS'20

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Modularity
Apply/extend the composition framework to more complex protocols and properties.

(e-voting protocols, forward secrecy for key-exchanges)

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Improve the automation,

- at the high-level, in the Squirrel Prover; (e.g. with SMT solvers)
- at the low-level, through SolvEq.

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Improve the automation,

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- at the low-level, through SolvEq.

Collaboration
There will not be one tool to rule them all. Use each for what it does best and combine formally the guarantees.

Privacy

What's next - in real life

Privacy

- It **matters**.

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- It **matters**.
- Most people, corporation and states **don't care about it**.

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I would like to try to do something about that...

Some appendixes

Composition on an example

Main intuition

Basic Theorem Example - Parallel Composition
Given two protocols $\mathcal{P}$ and $\mathcal{Q}$, with $\overline{n} = \mathcal{N}(P) \cap \mathcal{N}(Q)$, if:

- $\nu \overline{n}.\mathcal{Q}$ is $\mathcal{O}$-simulatable;
- $A \models \mathcal{P} \sim \mathcal{P}'$;
- the axioms A are sound for machines with access to $\mathcal{O}$.

Then $A \models \mathcal{P} \| \mathcal{Q} \sim \mathcal{P}' \| \mathcal{Q}$.

A small DDH example

Signed DDH

$A(a, skA)$ $\qquad\qquad$ $B(b, skB)$

$\xrightarrow{\text{sign}(g^a, skA)}$

$x_B = g^a$

$\xleftarrow{\text{sign}(<g^a, g^b>, skB)}$

$x_A = g^b$

$\xrightarrow{\text{sign}(<g^a, g^b>, skA)}$

$k_A = x_A^a$ $\qquad\qquad$ $k_B = x_B^b$

A small DDH example

The security property:

$$\|^{i \leq N}(A(a_i, skA); \textbf{out}(k_A) \| B(b_i, skB); \textbf{out}(k_B))$$

$$\sim$$

$$\begin{aligned}
&\|^{i \leq N-1}(A(a_i, skA); \textbf{out}(k_A) \| B(b_i, skB); \textbf{out}(k_B)) \\
&\| \;\; A(a_N, skA);\ \textbf{if}\ x_A = g^{b_N}\ \textbf{then out}(k_{N,N}) \\
&\qquad\qquad\qquad \textbf{else if}\ x_A \notin \{g^{b_i}\}_{1 \leq i \leq N}\ \textbf{then}\ \perp \\
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. We can simply prove:

$$
\begin{array}{c}
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$\hookrightarrow$ How to simulate the $N-1$ sessions ?

Simulating the sessions

What must the attacker be able to produce ?
He must be able to start some A:

$$\forall 1 \leq i \leq N-1.\ \mathrm{sign}(g^{a_i}, skA)$$

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Give the attacher access to $\mathcal{O}^{sign}_{T,skA}$ and $\mathcal{O}^{sign}_{T,skB}$ with:

$$T(m) = \text{true} \Leftrightarrow \exists 1 \leq i \leq N-1, r. \begin{cases} m = g^{a_i} \\ m =< g^{a_i}, r > \\ m =< r, g^{b_i} > \end{cases}$$

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$\hookrightarrow$ How to make the proof for such attackers ?

Generic axioms

T-EUF-CMA

For any computable function T, for all terms t such that sk only appears in key position:

$$\begin{aligned}&\texttt{checksign}(t, pk(sk))) \Rightarrow\\&\quad T(\texttt{getmess}(t))\\&\quad \bigvee_{\mathsf{sign}(x,sk)\in \texttt{St}(t)} (t \doteq \mathsf{sign}(x, sk)))\\&\sim \mathit{true}\end{aligned}$$

The final proof

Assumption

$$\mathtt{checksign}(t, pk(sk)) \Rightarrow$$
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\end{aligned}$$

$$\wedge \quad DDH : g^{a_N}, g^{b_N}, g^{a_N b_N} \sim g^{a_N}, g^{b_N}, k_{N,N}$$

The final proof

Goal

$$A(a_N, skA); \mathbf{out}(k_A) \| B(b_N, skB); \mathbf{out}(k_B)$$
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$$\parallel B(b_N, skB);\ \textbf{if } x_B = g^{a_N} \textbf{ then out}(k_{N,N})$$
$$\textbf{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \textbf{ then } \bot$$

The final proof

Synchronization

$$
\begin{aligned}
& A(a_N, skA);\ \text{if } x_A = g^{b_N} \text{ then out}(g^{a_N b_N}) \\
& \qquad \text{else if } x_A \notin \{g^{b_i}\}_{1 \leq i \leq N} \text{ then } \text{out}(x_A^{a_N}) \\
\| \ & B(b_N, skB);\ \text{if } x_B = g^{a_N} \text{ then out}(g^{a_N b_N}) \\
& \qquad \text{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \text{ then out}(x_B^{b_N})
\end{aligned}
$$

$$\sim$$

$$
\begin{aligned}
& A(a_N, skA);\ \text{if } x_A = g^{b_N} \text{ then out}(k_{N,N}) \\
& \qquad \text{else if } x_A \notin \{g^{b_i}\}_{1 \leq i \leq N} \text{ then } \bot \\
\| \ & B(b_N, skB);\ \text{if } x_B = g^{a_N} \text{ then out}(k_{N,N}) \\
& \qquad \text{else if } x_B \notin \{g^{a_i}\}_{1 \leq i \leq N} \text{ then } \bot
\end{aligned}
$$

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3. similar to (2);
4. similar to (2);

Formal composition theorems

A core theorem

Composition without replication

Let $C[\_{1}, \ldots, \_{n}]$ be a context such that the variable k_i is bound in each hole $\_i$ and $P_1(x), \ldots, P_n(x)$ be parametrized protocols, such that all channels are disjoint. Given an oracle $\mathcal{O}$, with $\overline{n} \supset \mathcal{N}(C) \cap \mathcal{N}(P_1, \ldots, P_n)$, if, with $k'_1, \ldots, k'_n$ fresh names,

1. $C[\mathbf{out}(1, k_1), \ldots, \mathbf{out}(n, k_n)] \cong_{\mathcal{O}} C[\mathbf{out}(1, k'_1), \ldots, \mathbf{out}(n, k'_n)]$
2. $\nu\overline{n}.\mathbf{in}(x).P_1(x) \| \ldots \| \mathbf{in}(x).P_n(x)$ is $\mathcal{O}$-simulatable

Then $C[P_1(k_1), \ldots, P_n(k_n)] \cong_{\mathcal{O}} C[P_1(k'_1), \ldots, P_n(k'_n)]$

A core theorem

Unbounded parallel Composition

Let $\mathcal{O}_r$ be an oracle and Ax a set of axioms both parametrized by a sequence of names $\overline{s}$. Let $\overline{p}$ be a sequence of shared secrets, $P(\overline{x})$, $R(\overline{x}, \overline{y}, \overline{z})$ and $Q(\overline{x}, \overline{y})$ be parametrized protocols. If we have, for a sequence of names $\overline{lsid}$ and any integers n, if with $\overline{s} = \overline{lsid}_1, \dots, \overline{lsid}_n$ n copies of $\overline{lsid}$:

1. $\forall\ 1 \leq i \leq n, \nu\overline{p}.t_{R(\overline{p},\overline{lsid}_i,\overline{s})}$ is $\mathcal{O}_r$ simulatable.
2. Ax is $\mathcal{O}_r$ sound.
3. $Ax \models t_{P(\overline{p})} \sim t_{Q(\overline{p},\overline{s})}$

Then, for any integer n:

$$\begin{aligned} &P(\overline{p}) \| \ !_n R(\overline{p}, \overline{lsid}, \overline{s}) \\ &\quad \cong Q(\overline{p}, \overline{s}) \| \ !_n R(\overline{p}, \overline{lsid}, \overline{s}) \end{aligned}$$

A core theorem

Unbounded parallel Composition
Let $\mathcal{O}_r$ be an oracle and Ax a set of axioms both parametrized by a sequence of names $\overline{s}$. Let $\overline{p}$ be a sequence of shared secrets, $P(\overline{x}, \overline{y})$ and $Q(\overline{x}, \overline{y}, \overline{z})$ be parametrized protocols. If we have, for sequences of names $\overline{lsid}_p, \overline{lsid}_q$ and any integers n, if with $\overline{s} = \overline{lsid}_{p,1}, \dots, \overline{lsid}_{p,n}, \dots, \overline{lsid}_{q,n}$ sequences of copies of $\overline{lsid}_p, \overline{lsid}_q$

1. $\forall\ 1 \leq i \leq n, \nu\overline{p}.t_{P(\overline{p}, \overline{lsid}_{p,i})}$ is $\mathcal{O}_r$ simulatable.
2. $\forall\ 1 \leq i \leq n, \nu\overline{p}.t_{Q(\overline{p}, \overline{lsid}_{q,i}, \overline{s})}$ is $\mathcal{O}_r$ simulatable.
3. Ax is $\mathcal{O}_r$ sound.
4. $Ax \models t_{P(\overline{p}, \overline{lsid}_p)} \sim t_{Q(\overline{p}, \overline{lsid}_q, \overline{s})}$

Then, for any integers n:

$$!_n P(\overline{p}, \overline{lsid}_p) \cong_{\mathcal{O}} !_n Q(\overline{p}, \overline{s}, \overline{lsid}_q)$$